

Exercise 6: Invariant density

We ask you:

Exercise: Change the initial distribution in:

Code 2.8 Logistic map: density, M trajectories.

Solution: In the Code 2.8, the initial points are selected uniformly in the interval $(0, 1)$, in the code below, the interval is $(0.5, 0.6)$. Now the line for the initial points is `xiniz0[1] <- runif(1,0.5,0.6)`.

```
# Logistic map: density, M trajectories, initial distribution: uniform in (0.5,0.6)
f.x<- function(x,r){
  r*x*(1-x)
}
r<- 4
M<- 1000 # number of trajectories
nstep<- 25 # number of iterations
xt<- numeric()
xiniz0<- numeric()
xens<-matrix(M,nstep) # to memorize the single trajectory
set.seed(1) # seed of the sequence of (pseudo) random numbers
for(l in 1:M){ # starting loop on the trajectories
  # it is possible to change the initial distribution
  xiniz0[l]<- runif(1,0.5,0.6)          # uniform distribution in (0.5,0.6)
  x<- xiniz0[l]
  xt[1]<- x
  for(i in 1:nstep){ # starting loop on the iterations
    y<- f.x(x,r)
    x<- y
    xt[i]<- x
    xens[l,i]<- xt[i]
  } # ending loop on the iterations
} # ending loop on the trajectories
mstep1<-1
mstep2<-2
mstep3<-3
mstep4<-5
mstep5<-nstep
lbin<- 0.02
windows()
par(mfrow=c(3,3),cex.main=0.8)
hist(xiniz0,probability=T,xlab="x(t)",ylab="Density",main="Initial distr.",
xlim=c(0,1),ylim=c(0,10),br=seq(0,1,by=lbin),col="black",border="black")
hist(xens[,mstep1],probability=T,xlab="x(t)",ylab="Density",main="t = 1",
xlim=c(0,1),ylim=c(0,40),br=seq(0,1,by=lbin),col="black",border="black")
hist(xens[,mstep2],probability=T,xlab="x(t)",ylab="Density",main="t = 2",
xlim=c(0,1),ylim=c(0,20),br=seq(0,1,by=lbin),col="black",border="black")
```

```

hist(xens[,mstep3],probability=T,xlab="x(t)",ylab="Density",main="t = 3",
xlim=c(0,1),ylim=c(0,10),br=seq(0,1,by=1bin),col="black",border="black")
hist(xens[,mstep4],probability=T,xlab="x(t)",ylab="Density",main="t = 5",
xlim=c(0,1),ylim=c(0,10),br=seq(0,1,by=1bin),col="black",border="black")
hist(xens[,mstep5],probability=T,xlab="x(t)",ylab="Density",main="t = 25",
xlim=c(0,1),ylim=c(0,10),br=seq(0,1,by=1bin),col="black",border="black")

```

The result is reported in Fig. 1. Notice the different scales in the histograms with $t = 1$ and $t = 2$. In the present case, we see that the con-

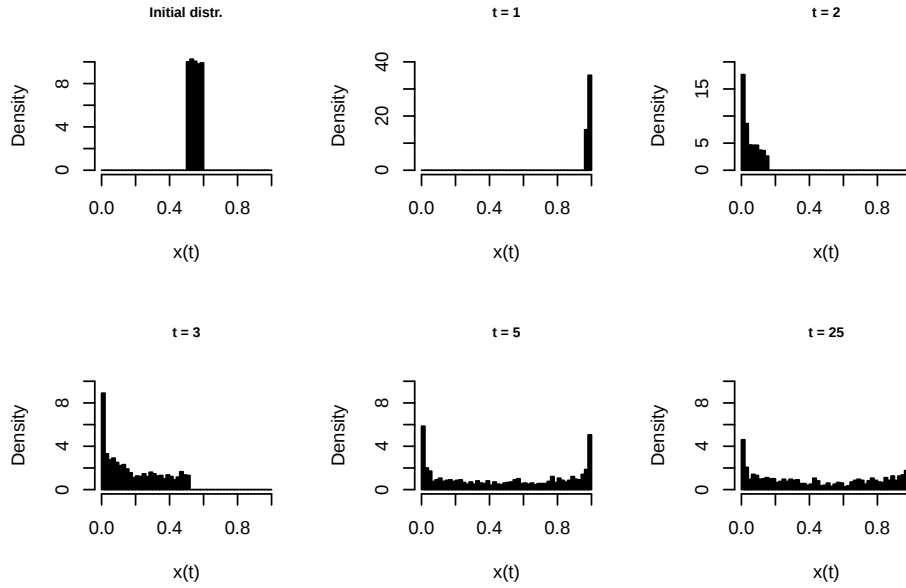


Figure 1 Evolution of density of an ensemble of 1000 trajectories of the logistic map with $r = 4$. Initial density uniformly distributed in $(0.5, 0.6)$.

vergence to the $Beta(0.5, 0.5)$ is slower with respect to the initial density uniformly distributed in $(0, 1)$ (Fig. 2.30).

We can change further the initial condition with the instruction:

```
xiniz0[1]<- rnorm(1,0.5,0.05)    # normal distribution
```

The initial points are sampled from a normal distribution with mean = 0.5 and standard deviation = 0.05. The result is reported in Fig. 2. Notice the different scales in the histograms with $t = 1$ and $t = 2$. Also in this case, the convergence to the $Beta(0.5, 0.5)$ is slower with respect to the initial density uniformly distributed in $(0, 1)$ (Fig. 2.30). The reader can vary the initial condition and also the value of r in the logistic map.

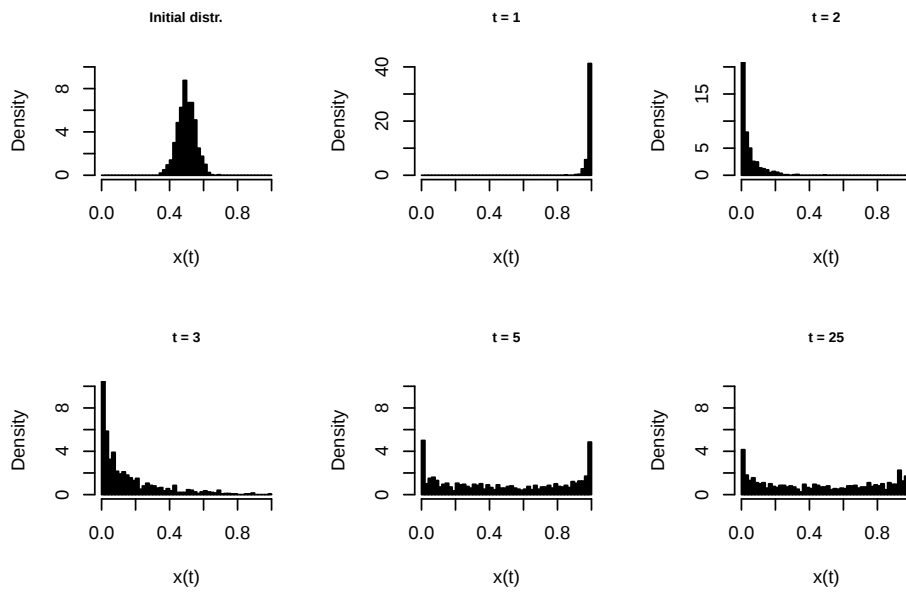


Figure 2 Evolution of density of an ensemble of 1000 trajectories of the logistic map with $r = 4$. The initial density is a normal with mean = 0.5 and standard deviation = 0.05.